des Zellkernes stark ab, wie aus Abbildung 7 deutlich erkennbar ist.

Aus diesen Befunden lässt sich die oben gemachte Annahme weitgehend begründen, dass die morphologischen Kernänderungen, der Umsatz des Kernes und des Nukleolus sowie schliesslich auch der Wirkungsgrad des Kernes und des Nukleolus, worauf ausdrücklich von HÄMMERLING[54] hingewiesen wird, vom Zytoplasma aus über die Menge der energiereichen Phosphorverbindungen gesteuert wird. Anderseits ist, wie zahlreiche Versuche erkennen lassen, die Kerntätigkeit Voraussetzung eines normalen Stoffwechsels des Zytoplasmas. Dies lässt sich deutlich an Zellen demonstrieren, bei denen der Kern entfernt wird. Im Verhältnis zu den kernhaltigen Zellen werden in den kernlosen Teilen früher oder später viele Syntheseprozesse reduziert, was schliesslich zum Tode dieser Teile führt[55]. Die Befunde zeigen, dass innerhalb der Zelle eine enge Wechselwirkung zwischen Kern und Zytoplasma besteht: der Kern ist von der Energie abhängig, die ihm durch die Mitochondrien zur Verfügung gestellt wird, während anderseits die Vorgänge des Zytoplasmas die Tätigkeit des Kernes benötigen. Für eine eingehende Darstellung dieser Kreisprozesse sei auf HÄMMERLING[54] verwiesen.

[54] J. HÄMMERLING, Rapp. Comm. 8me Congr. intern. Bot., Paris 1954 (im Druck).

[55] J. HÄMMERLING, Naturwissenschaften *33*, 337 und 361 (1946); Rev. Cytology *2*, 475 (1953). – J. BRACHET, *The Nucleic Acids II* (Acad. Press, New York 1955), S. 476.

Summary

In previous years the composition and function of chromosomes have been analyzed to a considerable extent, in contrast to the small number of investigations that have been made on nucleoli. There exist at present many conflicting opinions about the structure and function of the nucleolus, and therefore a critical survey of the facts and theories about the nucleolar substance appears necessary.

The structure, chemical composition and metabolism of the nucleoli of various cells are described in this publication. The regulation of the size and function of the nucleolus is also discussed.

It is concluded that the size and structure of the nucleolus is organ- and species-specific. A rapid synthesis of RNA and proteins occurs in the nucleolus. After a short interval, this disappears; and it is therefore concluded that the nucleolus must have a high turnover of these substances. The size and function of the nucleoli depends on the energy state of the cytoplasm.

It appears that the nucleolus can synthesize proteins which can be utilized by the chromosomes during their genetic function.

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A Closed Expression for Certain Probabilities in Wilcoxon's two Sample Test

Consider two samples of sizes n_1 and n_2 respectively, drawn from two univariate distributions. Call a value observed in the first sample a 1-value, and a value observed in the second sample a 2-value. Suppose that no 1-value equals any 2-value (when the underlying distributions are continuous, the probability of this occurrence is one). Form all possible pairs consisting of one 1-value and one 2-value. There are $n_1 n_2$ such pairs. Call those pairs among them in which the 1-value is smaller than the 2-value 1-2-pairs, and the pairs in which the 2-value is smaller than the 1-value 2-1-pairs. Denote the number of 1-2-pairs by U_{12} and the number of 2-1-pairs by U_{21}. Then:

$$U_{12} \geqq 0, \quad U_{21} \geqq 0, \quad \text{and } U_{12} + U_{21} = n_1 n_2.$$

If one repeatedly draws two samples of sizes n_1 and n_2 from two (univariate) distributions, one will not find the same number of 1-2-pairs with each repetition. Let $P(U_{12} = u,; n_1, n_2)$ denote the probability that the number of 1-2-pairs to be formed from two samples of sizes n_1 and n_2 is u, this probability being understood to be calculated under the hypothesis tested by Wilcoxon's two sample test (of which hypothesis the hypothesis that the $N = n_1 + n_2$ values are independently drawn from two identical distributions is a particular case). After this definition the definition of $P(U_{21} = u; n_1, n_2)$, $P(U_{12} \leqq u; n_1, n_2)$ and $P(U_{21} \leqq u; n_1, n_2)$ will be immediately clear. The last-named probability has been tabulated (in a triple-entry table of course) by MANN and WHITNEY[1] for $n_2 \leqq n_1 \leqq 8$, by VAN DER VAART[2]

[1] H. B. MANN and D. R. WHITNEY, Ann. Math. Stat. *18*, 50 (1947).

[2] H. R. VAN DER VAART, *Gebruiksaanwijzing voor de tests van Wilcoxon*, Rapport S 32 (M4) van de Statistische Afdeling van het Mathematisch Centrum, Amsterdam, with tables (1950), obtainable from the Mathematisch Centrum, Amsterdam; *cf.* the more detailed Rapport S 176 (M 65) van de Stat. Afd. v. h. Math. Centr., obtainable

(with more significant figures) for $n_2 \leqq n_1 \leqq 10$. Other tables are those by FESTINGER[3], by WHITE[4], and by AUBLE[5], which, however, are found to have been criticized for containing more or less serious errors. Using generating functions KEMPERMAN[6] has indicated a method by which to calculate the above probabilities by means of a double-entry table instead of a triple-entry table; he gave such a double-entry table covering the cases for which $u \leqq 40$ and $n \leqq 5$ [n = Min (n_1, n_2), the size of the smallest sample]. Essentially the same method was quite recently re-discovered (without the use of generating functions) and set forth in a detailed paper by FIX and HODGES[7] who gave the double-entry table needed in case $u \leqq 100$ and $n \leqq 12$. Some old work on partition theory, to be discussed in another paper, gives extensive tables closely connected with the last-mentioned two tables: EULER (1748, 1750) covering the cases $u \leqq 69$ and $n \leqq 11$, and $u \leqq 59$ and $n \leqq 20$, respectively, and MARSANO (1870) covering the case $u \leqq 103$ and $n \leqq 102$.

Now, in some other work on partition theory, BRIOSCHI[8], we found a result, dispensing with tables altogether, which we traduct here in terms of Wilcoxon probabilities. The result is interesting because it answers for the two sample problem a question left open by KRUSKAL[9] with respect to the several sample problem in general: namely, the question of a closed expression for certain probabilities in rank tests for the k ($\geqq 2$) sample problem. Moreover, the result allows one to calculate the exact probabilities in virtually all cases not covered by the tables one may have at hand: in fact, no table is needed at all – though for u large the calculations may turn out to be somewhat cumbersome.

The result, then, is this:

If k is a positive integer as well as i, define

$$\delta(k/i) = 1 \text{ if } k/i \text{ is an integer, and } 0 \text{ if } k/i \text{ is not.}$$

Furthermore define

$$s_k = \sum_{i=1}^{n_2} i \cdot \delta(k/i) - \sum_{i=1}^{n_2} (n_1+i) \cdot \delta(k/(n_1+i)).$$

It is clear that interchanging n_1 and n_2 leaves the value of s_k unaltered; hence if $n_2 < n_1$ the version here given will be chosen; if $n_2 > n_1$ one will prefer to interchange n_1 and n_2 in the expression for s_k. Finally define

$$\Delta(s, 0) = 1, \quad \Delta(s, 1) = s_1,$$

$$\Delta(s, u) = \begin{vmatrix} s_1 & -1 & 0 & \cdots & 0 \\ s_2 & s_1 & -2 & \cdots & 0 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdots & \cdot\cdot \\ s_{u-1} & s_{u-2} & s_{u-3} & \cdots & -(u-1) \\ s_u & s_{u-1} & s_{u-2} & \cdots & s_1 \end{vmatrix} \quad \text{for } u \geqq 2,$$

and

$$s^*_k = 1 + s_k, \quad {}^*s_k = -1 + s_k,$$

while the definition of

$$\Delta(s^*, u) \quad \text{and} \quad \Delta({}^*s, u)$$

be identical with that of $\Delta(s, u)$ except for the symbol s being replaced by s^* and $^*s$, respectively. Then if u be smaller than $\frac{1}{2}n_1n_2$:

$$P(U_{12} = u; n_1, n_2) = P(U_{21} = u; n_1, n_2) = (n_1!\, n_2!/N!\, u!)\, \Delta(s, u),$$
$$P(U_{12} \leqq u; n_1, n_2) = P(U_{21} \leqq u; n_1, n_2) = (n_1!\, n_2!/N!\, u!)\, \Delta(s^*, u),$$

while if in addition $u \geqq 1$:

$$P(U_{12} = u; n_1, n_2) - P(U_{12} = u-1; n_1, n_2) = (n_1!\, n_2!/N!\, u!)\, \Delta({}^*s, u).$$

For example $P(U_{12} \leqq 7; 49, 2) = 0{\cdot}015686$,
$P(U_{12} \leqq 23; 21, 5) = 0{\cdot}028717$.

A self-contained, mainly expository paper with proofs and details on applications is being prepared. Research on other connections between rank tests and the theory of partitions is in progress.

H. R. VAN DER VAART

Zoological Laboratory of the University, Leiden, August 23, 1955.

Zusammenfassung

Gewisse Wahrscheinlichkeiten, welche in der Anwendung von Wilcoxons Test für das Problem der zwei Stichproben wichtig sind, werden mittels einer Determinante geschlossen ausgewertet. Diese Determinante wurde bereits von BRIOSCHI auf ein Problem in der Theorie der Partitionen von ganzen Zahlen angewandt.

from the same source and written by D. WABEKE and C. VAN EEDEN (1955), who give an additional table slightly extending the preceding ones.

[3] L. FESTINGER, Psychometrika *11*, 97 (1946).

[4] C. WHITE, Biometrics *8*, 32 (1952).

[5] D. AUBLE, Bull. Inst. educat. Res. Indiana Univ. *1*, No. 2, 39 (1953); cited after: E. FIX and J. L. HODGES, Jr., Ann. Math. Stat. *26*, 301 (1955).

[6] J. H. KEMPERMAN, *De verdelingsfunctie van het aantal inversies in de test van Mann en Whitney*, Rapport T.W. 7, March 1950, Mathem. Centr., Amsterdam.

[7] E. FIX and J. L. HODGES, Jr., Ann. Math. Stat. *26*, 301 (1955).

[8] F. BRIOSCHI, Ann. Sci. mat. fis. (compilati da B. TORTOLINI) *7*, 303 (1856).

[9] W. H. KRUSKAL, Ann. Math. Stat. *23*, 525, 536 (1952).

Electro-optical Shift

Little work appears to have been done on the properties of liquid dielectrics under high electrical fields (direct or alternating). The effect of an electrical field on viscosity[1], on refractive index[2], on internal friction[3] and on the optics[4] of the system has been studied. In conjunction with research on liquid crystals[5] and liquid dielectrics in progress in this laboratory, it was observed that a beam of white light passing through a liquid dielectric can be shifted in its path upon application of an electrical field. Since work, to date, is of a preliminary nature, the present disclosure must be limited in detail.

A typical cell used to study the electro-optical shift consisted of a 50 ml beaker into which two semi-circular metal electrodes are inserted and held rigidly in place. The electrodes in this study were machined from brass. These electrodes are of such a size that they fill the beaker with the exception of a spacing of 5 mm at the center of the beaker and are mounted so that their faces are parallel. The liquid dielectric under study is placed in the space between the electrodes. The spacing between the electrodes is not critical; the important factors controlling it are arcing and voltage supply.

[1] E. N. DA C. ANDRADE, Proc. roy. Soc. (London) [A] *215*, 36 (1952).

[2] M. PAUTHENIER, C. r. Acad. Sci. *178*, 1899 (1924).

[3] JAN VAN CALKER and B. AUBKE, Z. Physik *131*, 443 (1952).

[4] J. KERR, Phil. Mag. *50*, 337 (1875).

[5] *Shedding Light on Liquid Crystals*, Chem. Eng. News *32*, 2962 (1954).